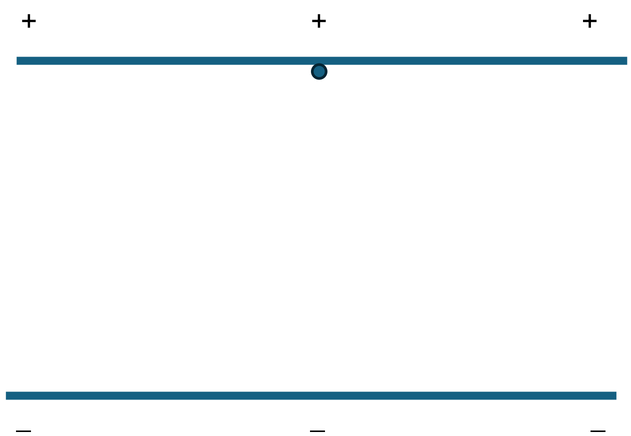
Motion in uniform fields worksheet

(a) Two parallel, oppositely charged plates are separated by a distance *d*. There is a potential difference *V* between the plates. A proton is placed at the positive plate.



The proton is released.

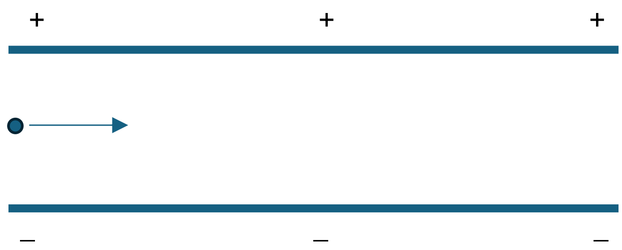
(i) State the kinetic energy *K* of the proton just before it impacts the negative plate.

(ii) Hence calculate the speed *v* of the proton just before it impacts the negative plate.

(iii) Show that the time *T* to reach the negative plate is given by $T = \sqrt{\dfrac{2m_p d^2}{eV}}$.

(iv) The proton is replaced by an alpha particle. State how *K*, *v* and *T* change, if at all.

(b) An electron enters the region between two parallel, oppositely charged plates with horizontal speed *u* midway between the plates. There is a potential difference *V* between the plates which are separated by a distance *d*. The length of the plates is *L*. The electron exits the plates without hitting the plates.



(i) Calculate the time taken for the electron to cover the horizontal distance L.

(ii) Calculate the vertical distance covered by the electron in the time found in (i).

(iii) Deduce that since the electron does not hit the plates $u^2 > \frac{eVL^2}{m_e d^2}$.

(iv) Determine the kinetic energy of the electron as it exits the plates.

(v) Hence deduce the speed of the electron as it exits the plates.

(vi) A magnetic field is established at right angles to the plane of the page. Determine the magnitude and direction of this field so that the electron goes through the plates without being deflected.

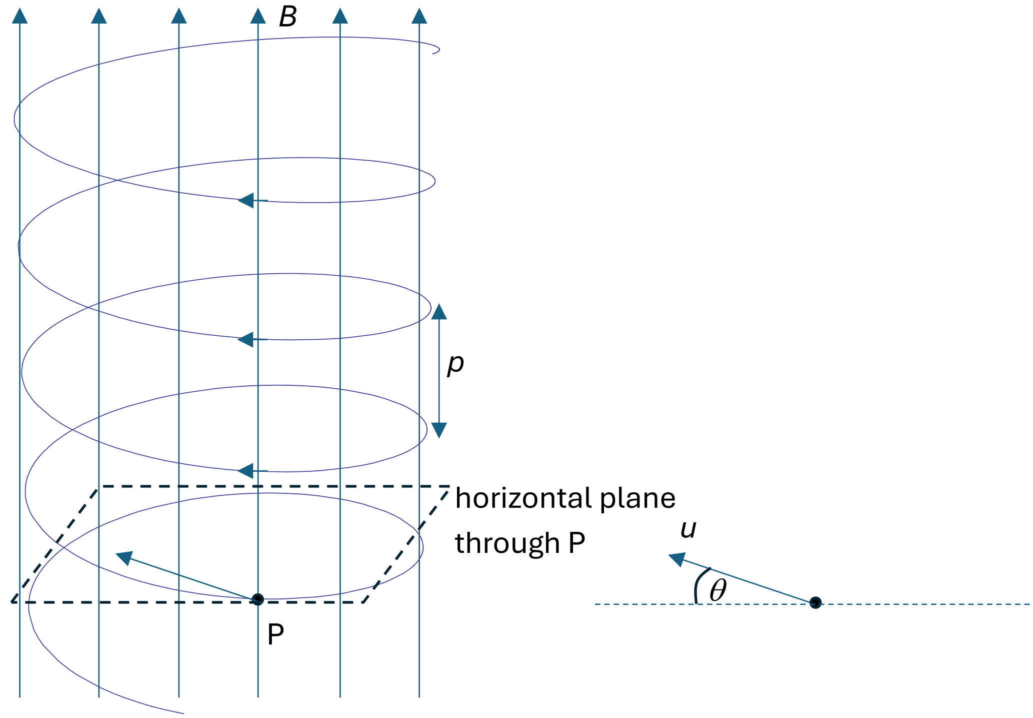
(c) A particle of mass m and charge q enters a region of magnetic field B. The velocity v of the particle is at right angles to the direction of the field.

(i) Explain why the speed of the particle does not change while in the region of magnetic field.

(ii) Show that the particle follows a circular path of radius R given by $R = \frac{mv}{qB}$.

(iii) Deduce that the period of revolution T is given by $T = \frac{2\pi m}{qB}$.

(d) A proton enters a region of uniform vertical magnetic field. The velocity of the proton at P makes an angle θ to the horizontal.

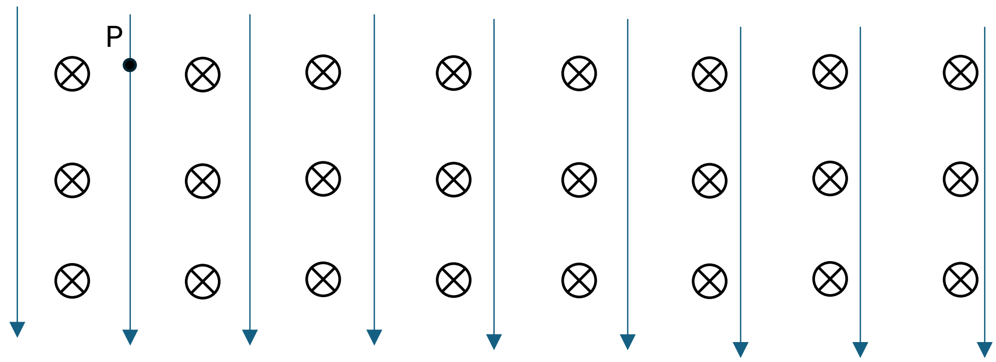


(i) State the direction of the magnetic force on the proton.

(ii) Explain why the motion of the proton is a spiral i.e. a combination of circular motion with speed $u\cos\theta$ and vertical uniform motion with speed $u\sin\theta$.

(iii) Deduce an expression for the distance *p*, i.e. the vertical distance travelled during one full revolution.

(e) A uniform magnetic field *B* is established into the plane of the page and a uniform electric field *E* is established on the plane of the page directed downwards. A proton is released from rest at point P at time *t* = 0.



(i) State the electric and magnetic force on the proton at $t = 0$.

(ii) On the diagram, draw a possible path of the proton after it is released.

(iii) Indicate on the path you have drawn the point at which the speed of the proton is a maximum.

Answers

(a)

(i) $K = eV = \frac{1}{2}m_p v^2$ and so $v = \sqrt{\frac{2eV}{m_p}}$

(ii) $d = \frac{1}{2}aT^2$ and $a = \frac{F}{m_p} = \frac{eE}{m_p} = \frac{eV}{m_p d}$. Hence $d = \frac{1}{2}\frac{eV}{m_p d}T^2$ and so $T = \sqrt{\frac{2m_p d^2}{eV}}$.

(iii) $K_\alpha = 2eV = 2K$. $v_\alpha = \sqrt{\frac{2\times 2eV}{4m_p}} = \frac{1}{\sqrt{2}}\sqrt{\frac{2eV}{m_p}} = \frac{1}{\sqrt{2}}v$. $T_\alpha = \sqrt{\frac{2\times 4m_p d^2}{2eV}} = T\sqrt{2}$.

(b)

(i) $t = \frac{L}{u}$.

(ii) $a = \frac{F}{m_p} = \frac{eE}{m_p} = \frac{eV}{m_p d}$. Hence $y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eV}{m_p d}(\frac{L}{u})^2$.

(iii) $y < \frac{d}{2}$ hence $\frac{1}{2}\frac{eV}{m_p d}(\frac{L}{u})^2 < \frac{d}{2}$. This gives $u^2 > \frac{eVL^2}{m_e d^2}$.

(iv) $K = \frac{1}{2}mu^2 + eV$.

(v) $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + eV$ hence $v = \sqrt{u^2 + \frac{2eV}{m}}$.

(vi) $eE = euB$. Hence $B = \frac{E}{u} = \frac{V}{ud}$.

(c)

(i) The magnetic force is at right angles to the velocity hence this force does no work. Hence the kinetic energy, and so speed, stays the same.

(ii) $qvB = \frac{mv^2}{R}$ hence $R = \frac{mv}{qB}$.

(iii) $T = \frac{2\pi R}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$.

(d)

(i) At right angles to both the velocity and the field i.e. horizontal and into the spiral.

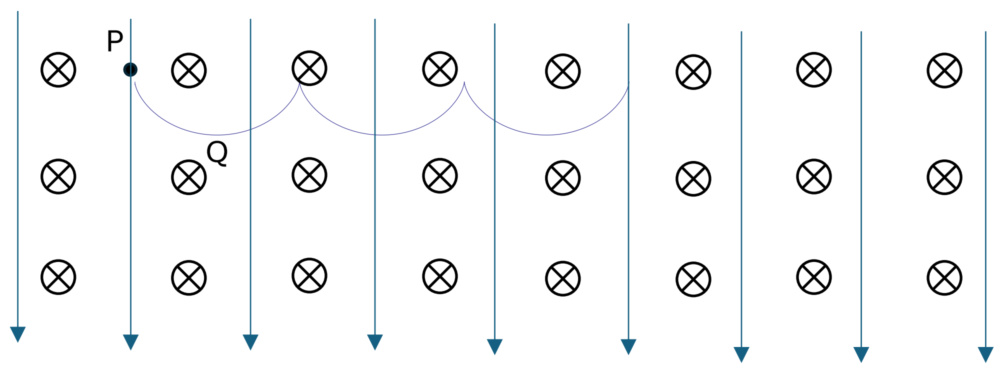
(ii) There is no force in the vertical direction so the particle moves with constant speed $u\sin\theta$ in the vertical direction. In the horizontal direction we have a centripetal force so circular motion with speed $u\cos\theta$.

(iii) $p = u\sin\theta \times T = u\sin\theta \dfrac{2\pi m}{qB}$.

(e)

(i) $F_e = qE$; $F_m = 0$.

(ii)



(iii) At the lowest point in the path such as Q in the diagram above.